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Mointoring Thickness Deviations in Planar Multi-Layered Elastic Structures Using Impedance Signatures

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Monitoring thickness deviations in planar multi-layered elastic structures using impedance signatures

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Abstract

In this letter, a low frequency ultrasonic resonance technique that operates in the (20 – 80 kHz) regime is presented that demonstrates detection of thickness changes on the order of +/- 10μm. This measurement capability is a result of the direct correlation between the electrical impedance of an electro-acoustic transducer and the mechanical loading it experiences when placed in contact with a layered elastic structure. The *relative* frequency shifts of the resonances peaks can be estimated through a simple one-dimensional transmission model. Separate experimental measurements confirm this technique to be sensitive to subtle changes in the underlying layered elastic structure.

PACS:

I. INTRODUCTION

The motivation for this letter is to report on a resonance technique that is useful for detecting material and geometrical variations in high contrast, lossy multilayered planar elastic structures. With this technique, subtle changes in the underlying structure can be monitored quantitatively with a single transducer by tracking the frequency shifts of the resonance peaks of the electrical impedance. A significant amount of research has been done in the area of electrical-mechanical impedance monitoring of elastic structures. The technique utilizes piezoelectric transducers that are physically attached to the elastic structure. Park *et al.*[1], has assembled an excellent overview and literature survey of the current state of impedance based health monitoring. In most cases these techniques rely on a comparison between an initial (pristine) state of the structure and corresponding measurements taken throughout the lifetime of the structure. The resulting differences are then used to infer qualitative states of damage or change in the health of the structure. Because the total electrical response (impedance) of the transducer is determined, in part, from the mechanical drive point impedance of the structure, changes in the underlying mechanical structure will alter the electrical impedance of the transducer. Thus, electrical impedance spectrograms can be utilized to infer changes in structural quantities, either geometrical or material provided a suitable model for the structure is developed.

II. THEORETICAL FORMULATION

The solution method approaches the problem by modeling the impedance response of the PZT transducer and the corresponding structure using a transmission matrix [2-7] Arbitrary backing and the load impedances can be applied at the mechanical terminals of the three-port network.

A. Total system impedance model

Following the convention presented in Reference 6, (Eq. 5.50 and 5.51), a generalized representation of a planar one-dimensional transducer model with arbitrary mechanical loading is given as

$$Z_{total} = \frac{V}{I} = \frac{\left[Z_{31} - \dfrac{Z_{32}Z_{21}}{Z_{back} + Z_{22}}\right]\left[-Z_{13} - \dfrac{Z_{13}Z_{21}}{Z_{back} + Z_{22}}\right]}{\left[Z_{load} + Z_{11} - \dfrac{Z_{12}Z_{21}}{Z_{back} + Z_{22}}\right]} + \left[Z_{33} - \frac{Z_{32}Z_{23}}{Z_{back} + Z_{22}}\right] \tag{1}$$

Here, Z_{total} is the impedance of the *entire* system, comprising the transducer and the planar multilayered elastic structure. The net effect of structure is represented by Z_{load}. Mechanical transducer characteristics are represented by Z_{back}, and the coefficients from the transmission matrix, are.

$$
\begin{aligned}
Z_{11} = Z_{22} &= A\sqrt{\rho_e c^D}\,\mathbf{cot}\left(\omega\sqrt{\frac{\rho_e}{c^D}}L\right), \\
Z_{12} = Z_{21} &= A\sqrt{\rho_e c^D}\,\mathbf{csc}\left(\omega\sqrt{\frac{\rho_e}{c^D}}L\right), \\
Z_{13} = Z_{31} = Z_{23} = Z_{32} &= \frac{e}{\omega\varepsilon^S}, \\
Z_{33} &= L/\omega\varepsilon^S A_o.
\end{aligned}
\tag{2}
$$

Where the density of the piezoelectric is, ρ_e , the elastic and piezoelectric stiffness coefficients are c^E and c^D, respectively. The piezoelectric field coefficient is, e, and the clamped (open circuit) dielectric constant is ε^S. The cross sectional area of the transducer is A and *L* is the thickness, and ω is the frequency. The mechanical input impedance, Z_{load}, is calculated from the impedance translation kernal[8] of Z_i^{local}, shown in Fig. 1.0. Each layer is represented by a known thickness, L_n, and an intrinsic specific impedance $\rho_n c_n$ based on layer density, ρ, and longitudinal sound speed, c. The load impedance is determined by starting with a known local impedance, Z^{known} at x=N and successively *translating* the calculated mechanical impedance through the entire structure to the x=L interface where the transducer layer is located. Thus, the total mechanical input impedance for the planar multilayered structure, $Z_L{}^{local}$ is equivalently the load impedance, Z_{load}. Equation 1 provides a concise method to estimate the impedance signature of a piezoelectric transducer mounted on a planar multilayered structure.

III. MODELING RESULTS

A numerical simulation was performed on the physical system described in Figure 2.0. Impedance signatures were generated at discrete frequencies from 10kHz through

100kHz. As a test to determine the sensitivity of the model to changes in layer thickness, five different impedance signatures were calculated for a range of thickness variations about a nominal acrylic layer thickness of L_{nom} = 23 mm. The variations are defined as, ($L_{acyrlic} = L_o + \delta$), where, $\delta_{1..N} =$ (– 0.051 mm to 0.051 mm) in steps of 0.025 mm. The intent is to investigate the sensitivity of the total impedance model to variations in the thickness of individual layers. The range of frequencies investigated spans several resonances of the transducer layer. It is near these resonant peaks of the transducer, that the impedance signatures are sensitive to subtle changes in the underlying planar structure. Figure 3 shows the magnitude of the *total* impedance Z_{total} as calculated from Eq.1. Resonances of the coupled transducer-structure are notated with Roman numerals. For each of the three boxed regions, an expanded view has been provided to illustrate the frequency shifts that occur as the thickness of the lower acrylic layer is varied. These regions were selected at the point of steepest ascent (i.e., $\max(dZ_{total}(\omega)/df)$), which defines locations along $|Z_{total}(\omega)|$ where the frequency shifts are most sensitive to structural variations. These regions also correspond to locations the in phase space where the phase changes sign, $\mathrm{sgn}(d\phi/df) = \pm 1$, also indicating a resonance condition. The expanded regions clearly illustrate measurable shifts in frequency of the transducers impedance due to small changes in the acrylic layer thickness. Table I summarizes a parametric study of the geometrical sensitivities by varying each elastic layer separately by $\delta_{1..N}$. The corresponding frequency shift at each resonance for a given layer change will be different, owing to the dynamic interaction between the different layers. This frequency selectivity provides a method to target individual layers by monitoring the resonance peaks with the greatest sensitivity to those layers.

IV. EXPERIMENTAL RESULTS

An experimental assessment of the detection sensitivity was conducted on two planar multilayered structures identical in diameter (30cm) and material composition to the previous numerical study with a slight thickness variation in two of the layers. Metrological differences between the disks A and B are listed in Table IV. The transducer is a simple PZT disk 30mm dia. x 2.76mm thick. The fundamental resonance mode for this thickness poled transducer is estimated at *58* kHz which represents a radial mode with uniform bowing through the thickness of the transducer[10]. A thin (0.25mm) layer of alcohol is used as a mechanical coupling layer between the transducer and the disks. The thickness of the coupling layer is maintained by a small plastic shim (0.25mm thick x30mm dia. x 2mm wide) that is attached to the PZT disk. An Agilent 4294 precision impedance analyzer is used to measure the electrical impedance signature from the PZT4 disk mounted in the center of the multilayered disks. The analyzer captured impedance measurements at 4000 discrete frequencies between 40kHz and 80kHz using a 1 volt drive signal.

A large planar geometry relative to the transducers size is chosen to minimize edge effects from the test object throughout the frequency band of operation and to emphasize quasi one-dimensional behavior through the thickness of the disks. The objective is to compare the *relative* frequency shifts at $\max(dZ_{total}(\omega)/df)$ estimated by the model and measured experimentally. The results for the experimental measurements and the one-dimensional planar predictions: in each layer, and the total are given in Table 1.0. The sign of the frequency shift indicates an increase or decrease in a layers thickness. For each experimental measurement, the transducer was removed and re-attached to the center of the disk. Ten separate measurements were recorded on each disk, from the ten measurements on each disk; an average frequency shift was measured. In this case, the predicted frequency shift (Δf_{mod}) from the model is 3.69 Hz and the experimental frequency shift (Δf_{exp}) is 3.28 +/-0.65 Hz. Variations in the mechanical coupling (bond) between the PZT disk and the structure for the measurements are the limiting factor for the determination of the smallest displacement which, in terms of experimental measurements, is on the order of 10μm.

The actual response (impedance signature) of the transducer used in the experiments, will be different from the modeled impedance signatures. This is expected due to the fact that the PZT transducer used in the experiments is not a one-dimensional layer, but a three-dimensional disk operating in a thickness mode resonance. However, the mechanical response (Z_{load}) of the planar disks are accurately represented using a one-dimensional approximation. Since the region under investigation, (e.g. multilayered planar disks) are effectively one-dimensional in thickness, the relative shifts in the model and the experiment will be in good agreement. Further refinements might include numerically fitting the model estimate of the transducer impedance to experimental measurements when the transducer is off the structure. A constrained optimization procedure could be used to adjust material parameters and loss coefficients and essentially calibrate the transducer model against experimental impedance measurements thus removing them as unknowns. Alternatively, the three dimensional complexities of the transducer could easily be included in the modeled estimates by incorporating finite elements. In either case, the transducer is simply a method to relate electrical quantities (frequency, voltage and current) to mechanical quantities (Z_{load}). Absolute comparisons between the model and experimental impedance signatures are not necessary.

Accurate interpretation of the frequency shifts of the resonance peaks will require prior knowledge about the multilayered system as found in design documents or manufacturing specifications, these in turn are incorporated into the one-dimensional model to guide the selection of an optimal transducer with impedance characteristics tuned for the particular layer under investigation. Because simultaneous variation of the layers in the structure can and will affect the resonant frequency shift in a complicated manner, this simple one-dimensional model can also be implemented in a Bayesian approach with measured impedance signatures to infer quantitative results regarding changes in the layers.

V. CONCLUSIONS

An approach has been developed for estimating the impedance shifts associated with changes in the geometry and elastic properties of a multilayered elastic structure. The simple examples have shown that resonances in the total system impedance are sensitive to small variations in layer thicknesses in the multilayered structure. The current one-dimensional model is sufficient for accurately predicting *relative* resonant frequency shifts of a piezoelectric transducer on planar structures.

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TABLE I. Predicted displacement sensitivities (mm/Hz) in each layer, and for each resonance.

Layer	L (mm)	ρ (gm/cm)	c (mm/µS)	Resonance I mm/Hz	Resonance II mm/Hz	Resonance III mm/Hz
PZT	5.76	4.32	4.50			
Alcohol	0.25	0.78	1.15			
Steel	2.76	7.70	5.90	9.34×10^{-3}	2.29 x 10-4	2.42×10^{-4}
Sylgard	0.23	1.12	1.03	2.10×10^{-4}	1.08×10^{-3}	6.63×10^{-5}
Acrylic	23.0	1.11	2.5	1.38×10^{-3}	1.08×10^{-3}	4.11×10^{-4}

TABLE II. Measurements and Model estimates of relative frequency shifts of resonance II, due to thickness differences.

Layer	Thickness [a] Disk A	Thickness [a] Disk B	Model Δf *(Hz)*	Measured [b] Δf *(Hz)*
Steel	2.760	2.760	0	
Sylgard	0.203	0.153	-46.29	
Acrylic	23.724	23.770	38.95	
Total frequency shift Δf of Resonance II due to changes in sylgard and Acrylic layers.			3.69	3.28

[a] all measurements in mm (+/- 0.001 mm)

[b] repeatability of 10 separate measurements (+/- 0.65 Hz)

Figure	Caption
1	Basic one-dimensional geometry and transducer orientation for the interrogation of a planar multilayered elastic structure. Each layer is described by a thickness, L, a density, ρ, and a longitudinal sound speed, c. The current into the transducer is I, and the drive voltage is V. The impedances, Z_{back} and Z_{load} represent either known impedances on the boundary or an intrinsic specific impedance for an unbounded region. At each interface, Z_n^{local} and Z_{n+1}^{local} denote the translated impedances through a given layer. The mathematical kernel for n^{th} layer is also shown[8]. The total mechanical input impedance for the multilayered elastic structure is Z_{load} and Z_o is the intrinsic impedance of the transducer.
2	The Numerical model for the impedance study. The transducer is modeled as a single air backed layer of PZT4 coupled to the upper surface of the elastic structure by a thin layer of alcohol. Here, Z_{load} is the *translated* input impedance beginning on the RHS at x = L_5 and stepping through each layer to the transducer to x = L_1. Materials are listed in Table I and Ref 9 and 10.
3	Frequency shifts of the *total* input impedance $Z_{total}(\omega)$ due to changes in the thickness of the lower acrylic layer. Five different impedance signatures were calculated based on a range of thicknesses for the acrylic layer ($L_{acry} = L_o + \delta$), where, $\delta =$ (– 0.051 mm to 0.051 mm) in steps of 0.025 mm. The nominal impedance signature (L_o = 23 mm , See Table I) is denoted with a heavy black line. The three resonances, notated by Roman numerals and zoom boxes indicate the regions where $\max(dZ/df)$ and therefore sensitive to structural variation.